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HEAT EMISSION OF SMALL PARTICLES

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The investigation of the heat emission of small particles has important value for geophysics, astrophysics and physical chemistry.

By converting the high frequency of solar radiation into the low frequency of heat, the particle mixture participates in a noticeable way in the heat balance in the lower layers of the atmosphere. In the upper layers where a large number of particles of dust bound by the meteoric flow, continuously enclosing earth, absorbing and emitting radiations by particles is one of the important mechanisms of controlling by heat processes.

Corresponding to Kirchkoff's law, the emission of particles is connected with that of absorption. To establish the corresponding formulas let us consider an infinitely large black interior sphere (radius R), in the center of which our particle is placed.

The stream energy from an infinitely small area ds which strikes the particle will be (B_λ is the intensity of the area).

$$dF_\lambda = B_\lambda \, ds \, \frac{\pi a^2}{R^2}$$

The share absorbed by the particles will be $\frac{k_n}{\pi a^2}$.[1]
In the same way, the particle, by the amount $\frac{k_n}{\pi a^2} dF_\lambda$, decreases the quantity

[1] $\frac{dF_\lambda}{\pi a^2}$ is the intensity of emission, k_n is the absorption coefficient, a is the radius of a particle.

of radiation which the area ds emits to the exact same area situated on the other end of a diameter.

In the condition of heat equilibrium this decrease of the stream will be compensated by the emission of particles on the same area. In the same way, the intensity of the particle $b_\lambda(T)$ will be

$$b_\lambda(T)\,\frac{ds}{R^2} = \frac{k_n}{\pi a^2}\,B_\lambda\,ds\,\frac{\pi a^2}{R^2}$$

$$b_\lambda(T) = k_n\,B_\lambda(T) \tag{1}$$

The intensity of a unit area of a particle B_λ^* will be πa^2 times less.

The emission of a particle is isotropic. The complete stream, emitted by a particle in unit time, will be

$$f_\lambda = 4\pi k_n\,B_\lambda\;(T) \tag{2}$$

For large black particles $k_n = \pi a^2$. In the same way

$$f_\lambda = 4\pi^2 a^2\,B_\lambda\;(T) = 4\pi^2 a^2\,B_\lambda \tag{3}$$

It is known that the total radiation of a plane area ds outwards equals πBds. In accordance with this the latter formula gives the total emission of a large black ball.

If the reflection of light is considered then, in formula (3) it is necessary to introduce the factor $(1-R^{(1)})$. Here $R^{(1)}$ is the share of the light stream reflected by the sphere (see (1) formula 28). For practical calculations of the emission of a large ball it is not at all necessary to make detailed calculations of the integral for $R^{(1)}$. With the accuracy of 3-4% it is sufficient, for example, to use the formula of Gauss in three ordinates.

Turning now to small particles. Here the general formula (1) produces a series of interesting consequences.

For small particles the decrease is considerably large dispersion.

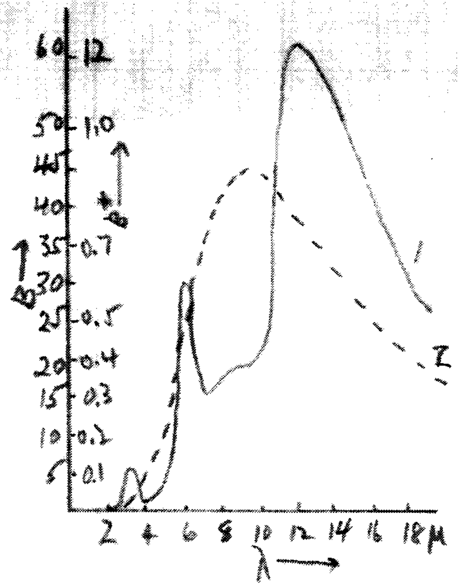

Fig. 1.

1. WATER DROP

2. BLACK BODY

$$b = \frac{36\pi n\kappa}{|m^2+2|^2} v \frac{2h}{c^3}\left(\frac{kT}{h}\right)^5 \int_0^\infty \frac{x^4 dx}{e^{x-1}}$$

There enters here the integral (let us denote it by α) easily calculated

$$\alpha = \int_0^\infty x^4 dx \ (e^{-x}+e^{-2x}+ \ldots) = \left(\sum_{k=1}^\infty \frac{1}{k^5}\right)\int_0^\infty e^{-t}t^4 dt$$

$$\int_0^\infty e^{-t}t^4 \, dt = \Gamma\ (5) = 24, \sum_{k=1}^\infty \frac{1}{k^5} = 1.0369..$$

([4] page 244).

In the same way,

$$\alpha = 24.8856$$

The intensity of a particle b, therefore, will be

$$b = \frac{b_o}{4\pi} \frac{n\kappa}{|m^2+2|^2} vT^5$$

$$b_o = \frac{72\pi\alpha k^5}{h^4 c^3}$$

The constant b_o is related to the Stefan-Boltzmann constant σ :

$$b_o = \gamma \frac{k}{ch} \sigma = 21.808\ldots .10^{-5} \frac{\text{erg}}{\text{deg}^5\text{cm}^3\,\text{sec}}$$

(γ is a multiplier = 5.7486)

The total emission of a particle is

$$f = b_o \frac{n\ \kappa}{|m^2+2|^2} v\ T^5$$

We obtained the important result: The integral emission of a small particle is proportional to the fifth degree of the temperature.[2]

In the case of small conducting particles (corresponding to the Hagens-Rubens law) the emission will be $\sim\sigma vT^6$. In the same way, the temperature dependence of emission is related to the temperature rate of resistance. For metals usually $\sigma\sim 1/T$, such that here $f\sim T^5$.

For lower temperatures (by comparison with Debye's temperature of a body)

[2] If $|m^2+2|$ is a constant.

the motion of σ is complex. The theory reduces here to the dependence $\sigma\sim\frac{1}{T^5}$. This means that the emission of small metallic particles with low temperatures will be $\sim T$.

By its electrical properties, dust probably is nearly always a semiconductor. For good conduction of semiconductors (electrons) $\sigma\sim e^{\Delta/T}$, the emission of small particles will be $\sim T^6 e^{\Delta/T}$.

For small particles (with constant m) the Winn law of mixing occurs. However, the constant in this law will be different from its usual value by a factor α/α^*. Here α and α^* are roots of the transcendental equation [3].

$$1 = \frac{\alpha}{5} + e^{-\alpha} \; ; \; 1 = \frac{\alpha^*}{6} + e^{-\alpha^*}$$

This factor $\frac{\alpha}{\alpha^*} = 0.8296...$

Particles, suspended in air, give up their heat by heat conduction and radiation. It is easy to see that the first flow is $k'/4aT^3\sigma$ greater than the second (k' is the coefficient of heat conduction of air; T is temperature). This quantity is $\sim 10^4$ in the lower layers and assumes the value ~ 1 only on the periphery of the atmosphere. In interstellar space radiation is the single principle emitter of heat. The equilibrium between absorption and emission defines the temperature of a particle found there.

An increased temperature of air (in unit time) connected by the direct heating of it at the expense of absorption of radiation by particle mixtures is determined by the quantity $\Delta T = I_0\pi a^2 n/c$ (n is the number of particles in $1 cm^3$, c is the heat capacity of $1\ cm^3$). This gives almost $0.05 - 0.1\ ^\circ/hr$.

Both specified estimates are related to large particles $(k_n = (\pi n^2)$. For small particles the relative role of emission will be considerably less.

The same relates to the heating of air by particles.

Let us note in conclusion that the emission of small particles to a considerable extent, defines the spectrum of flames.

REFERENCES

1. K. S. Shifrin: Isvestia - Series Geophys and Geogr. No. 2, 1950.

2. G. Mie: Ann. d. Phys. V. 25, 309 (1908)

3. P. Drude: Optics (1935)

4. I. M. Ryzhik: Tables of integrals, sums, series and derivaties, 1948.